

# Short-range Three-Nucleon Forces Effects on Nucleon-Deuteron Scattering

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Effects of three-nucleon forces arising from the exchange of a pion and a scalar-isoscalar object among three nucleons on nucleon-deuteron scattering observables are studied.

## 1. INTRODUCTION

The introduction of a three-nucleon force (3NF) arising from the exchange of two pions among three nucleons ( $2\pi E$ ) as shown in Fig. 1 (a) into nuclear Hamiltonian is known to get rid of discrepancies between experimental data and theoretical calculations with realistic two-nucleon forces (2NFs) for the three-nucleon (3N) binding energies and nucleon-deuteron (ND) differential cross sections. On the other hand, the  $2\pi E$  3NF is unsuccessful in explaining some ND polarization observables, *e.g.*, too small effects to vector analyzing powers (VAPs) and undesirable contributions to tensor analyzing powers (TAPs) in low-energy ND elastic scattering (see Fig. 2).

In Refs. [4,5], we have pointed out that tensor components in the  $2\pi E$  3NF should be responsible to the problem in TAPs. In this paper, we examine a 3NF due to the exchange of a pion and a scalar-isoscalar object ( $\pi$ -S) shown in Fig. 1 (b) as a possible source of tensor interactions that have different characteristics from those of the  $2\pi E$  3NF.

After giving a general form of the  $\pi$ -S 3NF in Sec. 2, numerical results for the 3N binding energy and ND scattering observables will be presented in Sec. 3. Summary is given in Sec. 4.

## 2. PION-”SCALAR-ISOSCALAR-OBJECT” EXCHANGE THREE-NUCLEON FORCES

We will consider the following models for the  $\pi$ -S 3NFs:  $\pi$ - $\sigma$  exchange with the excitation of the Roper resonance  $N^*(1440)$  ( $(\pi\sigma)_N$ ) [6,7];  $\pi$ - $\sigma$  exchange corresponding to the nucleon Born diagrams (so called pair or Z diagrams) for the PV ( $(\pi\sigma)_{Z,PV}$ ) or the PS ( $(\pi\sigma)_{Z,PS}$ )  $\pi NN$  coupling [6,7];  $\pi$ -”effective scalar field” exchange by a linear model [8]

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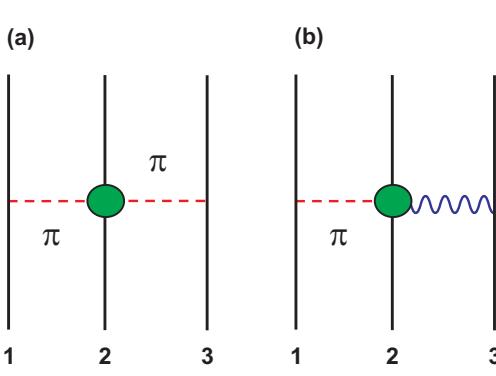


Figure 1. Diagrams for the  $2\pi$ E 3NF (a) and the  $\pi$ -S 3NF (b). The wavy line between the nucleons 2 and 3 represents the exchange of a scalar-isoscalar object.

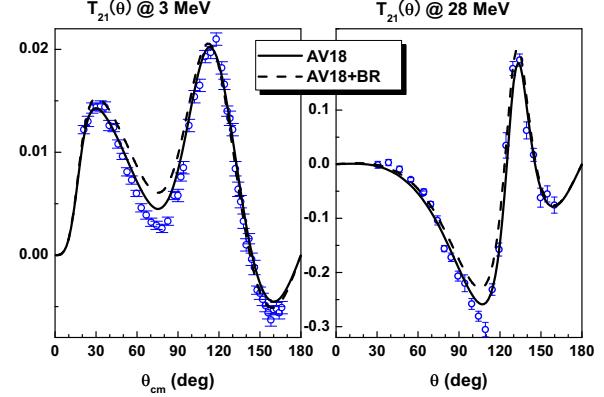


Figure 2.  $T_{21}(\theta)$  of  $pd$  elastic scattering at  $E = 3$  MeV and  $E = 28$  MeV. The data are taken from Refs. [1,2] for 3 MeV and from Ref. [3] for 28 MeV.

or by a nonlinear model [8];  $\pi$ -”effective  $2\pi$ ” exchange ( $(\pi-2\pi)$ ) [9]. A coordinate space representation of these potentials corresponding to diagram Fig. 1 (b) is:

$$W_{12,3}(\mathbf{r}_{12}, \mathbf{r}_{32}) = (\vec{\tau}_1 \cdot \vec{\tau}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla}_{12}) \left[ V_a(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla}_{12}) + V_b(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla}_{32}) \right] Y_\pi(r_{12}) Y_\sigma(r_{32}), \quad (1)$$

where  $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$  with  $\mathbf{r}_k$  being a position vector of nucleon  $k$ . When a dipole  $xNN$  form factor with a cutoff mass  $\Lambda_x$  is used, the function  $Y_x(r)$  becomes  $Y_x(r) = e^{-m_x r}/r - \{1 + (\Lambda_x^2 - m_x^2)r/(2\Lambda_x)\}e^{-\Lambda_x r}/r$ . Expressions of  $V_a$  and  $V_b$  for the  $\pi$ -S 3NFs [6,7,8,9] are shown in Table 1.

Table 1

The potential strengths  $V_a$  and  $V_b$  for the  $\pi$ -S 3NF models and and effects of the 3NFs on the  $^3\text{H}$  energy:  $\Delta E_{3NF} \equiv E_{3,\text{AV18+3NF}} - E_{3,\text{AV18}}$ .

	$V_a$	$V_b$	$\Delta E_{3NF}$ (MeV)
$(\pi-\sigma)_{N^*}$	$\frac{g_\pi g_\pi^*}{4\pi} \frac{g_\sigma g_\sigma^*}{4\pi} \frac{m_\pi^4}{2(m^* - m_N)m_N^2}$	—	-0.32
$(\pi-\sigma)_{Z,PV}$	—	$-\frac{g_\pi^2}{4\pi} \frac{g_\sigma^2}{4\pi} \frac{m_\pi^4}{4m_N^3}$	+0.47
$(\pi-\sigma)_{Z,PS}$	$\frac{g_\pi^2}{4\pi} \frac{g_\sigma^2}{4\pi} \frac{m_\pi^4}{4m_N^3}$	—	-1.04
Linear	$\left(\frac{g_\pi^2}{4\pi}\right)^2 \frac{m_\pi^4}{4m_N^3}$	—	-1.99
Nonlinear	$\left(\frac{g_\pi^2}{4\pi}\right)^2 \frac{m_\pi^4}{4m_N^3} \left(1 - \frac{m_N}{gf_\pi}\right)$	$-\left(\frac{g_\pi^2}{4\pi}\right)^2 \frac{m_\pi^4}{4m_N^3} \left(\frac{m_N}{gf_\pi}\right)$	+0.23
$\pi-2\pi$	$-\frac{g_\pi^2}{4\pi} \frac{g_s^2}{4\pi} \frac{m_\pi^4}{4m_N^3} \frac{m_N m_\pi}{3\alpha_{00}^+ f_\pi^2}$	$-\frac{g_\pi^2}{4\pi} \frac{g_s^2}{4\pi} \frac{m_\pi^4}{8m_N^3} \left(\frac{1}{3\alpha_{00}^+} \frac{m_N m_\pi}{f_\pi^2} + 1\right)$	+0.74

### 3. NUMERICAL RESULTS

In the present calculations, we use the following parameters:  $g_\pi^2/4\pi = 14.4$ ; another coupling constants from Table III of Ref. [7];  $f_\pi = 93$  MeV; a set of  $\{\alpha_{00}^+ = 3.68, g_s = 4.36, m_s = 393$  MeV} for the  $(\pi, 2\pi)$  3NF [9];  $\Lambda_\pi = 800$  MeV and  $\Lambda_\sigma = 1300$  MeV. First, we note that the Argonne  $V_{18}$  (AV18) 2NF [10] underbinds the triton ( ${}^3\text{H}$ ) by 0.85 MeV, and the Brazil  $2\pi\text{E}$  (BR<sub>800</sub>) 3NF [11,12] gives an additional attraction of -1.75 MeV. Thus a repulsive effect is expected to the  $\pi$ -S 3NF to complete the nuclear Hamiltonian.

Calculated values of the  ${}^3\text{H}$  energy for the AV18 plus each of the  $\pi$ -S 3NF models are presented in Table 1 as differences from the AV18 calculation. This shows that a  $\pi$ -S 3NF with (positive)  $V_a$ -term produces an attractive contribution to the  ${}^3\text{H}$  energy, and that with (negative)  $V_b$ -term a repulsive contribution, which is consistent with the results given in Ref. [7]. The  $(\pi-2\pi)$  3NF consists of a negative  $V_a$ -term and a negative  $V_b$ -term, and produces a repulsive effect mostly due to the  $V_a$ -term.

In order to investigate effects of each term in the  $\pi$ -S 3NF on ND scattering observables, we pick up the following two  $\pi$ -S 3NF models to reproduce the  ${}^3\text{H}$  energy together with the AV18 2NF, the BR<sub>800</sub> 3NF, and a phenomenological spin-orbit type 3NF (SO) [13]:

- The  $(\pi-\sigma)_{Z,PV}$  3NF as a representative to  $V_b$  term;
- The  $(\pi-2\pi)$  3NF as a representative to  $V_a$  term although it includes a small effect from  $V_b$  term. (In this case, we take  $\Lambda_\sigma = 800$  MeV.)

The inclusion of the SO 3NF is effective to reproduce the VAPs, but gives only minor effects on the  ${}^3\text{H}$  energy and the TAPs at low energies.

Numerical results of Faddeev calculations [14] for the tensor analyzing power  $T_{21}(\theta)$  at  $E = 3$  MeV and  $E = 28$  MeV are presented in Fig. 3, where we plot the experimental data and the results with the 3NFs divided by the AV18 calculations.

For  $T_{21}(\theta)$  at 3 MeV, both  $\pi$ -S 3NFs equally tend to cancel the effect due to the  $2\pi\text{E}$  3NF opposite to the data, but still leave an amount of discrepancy.

At 28 MeV, the 3NFs contribute to  $T_{21}(\theta)$  in different ways depending on scattering angles. At scattering angles of  $50^\circ$  to  $80^\circ$ , the calculation with the BR<sub>800</sub> 3NF and the one with the BR<sub>800</sub> +  $(\pi, 2\pi)$  3NFs look consistent with the data. On the other hand, at scattering angles of  $90^\circ$  to  $120^\circ$ , the calculation with the BR<sub>800</sub> +  $(\pi-\sigma)_{Z,PV}$  3NFs as well as the one with only 2NF look consistent with the data.

In Fig. 4 (a), we display results of the transversal  $\Delta\sigma_T$  and the longitudinal  $\Delta\sigma_L$  asymmetries of the spin dependent total cross sections in  $\vec{n} - \vec{d}$  scattering comparing with recent experimental data of  $\Delta\sigma_L$  [15]. In Ref. [16], we showed that effects of tensor interactions are prominent in the difference of  $\Delta\sigma_T - \Delta\sigma_L$ . As expected, differences of tendency in tensor components of the 3NFs are significantly observed in Fig. 4 (b).

### 4. SUMMARY

We have examined three-nucleon forces arising from the exchange of  $\pi$  and scalar-isoscalar object among three nucleons:  $\pi$ - $\sigma$  exchange via Z-diagram ( $V_b$ -term in Eq. (1));  $\pi$ -effective  $2\pi$  exchange ( $V_a$ -term, small  $V_b$ -term). Both models produce repulsive effects on the  ${}^3\text{H}$  energy to compensate strong attraction caused by the  $2\pi\text{E}$  3NF. Each 3NF affects polarization observables of nucleon-deuteron scattering,  $T_{21}(\theta)$  and  $\Delta\sigma_T - \Delta\sigma_L$ ,

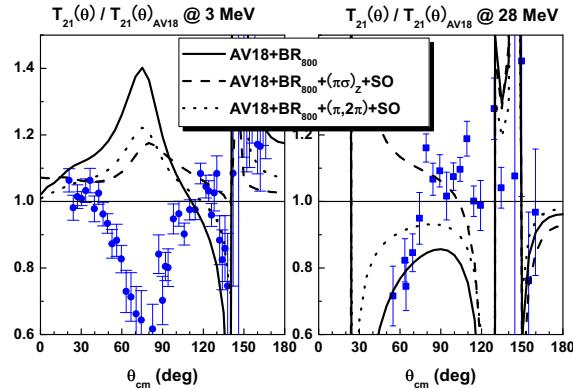


Figure 3.  $T_{21}(\theta)$  of  $pd$  elastic scattering normalized by the AV18 calculation at  $E = 3$  MeV and  $E = 28$  MeV. The data are taken from Refs. [1,2] for 3 MeV and from Ref. [3] for 28 MeV.

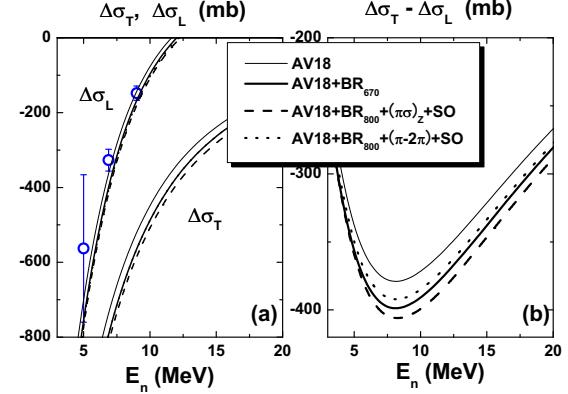


Figure 4. Spin-dependent total cross sections  $\Delta\sigma_T$  and  $\Delta\sigma_L$  (a), and their difference (b). The data of  $\Delta\sigma_L$  are taken from Ref. [15].

in different ways. This shows that further measurements of these observables at some energies provide additional information on three-nucleon forces, which should be included in the nuclear Hamiltonian in addition to the  $2\pi E$  3NF.

## REFERENCES

1. K. Sagara, *et al.*, Phys. Rev. C **50** (1994) 576.
2. S. Shimizu, *et al.*, Phys. Rev. C **52** (1995) 1193.
3. K. Hatanaka, *et al.*, Nucl. Phys. **A426** (1984) 77.
4. S. Ishikawa, M. Tanifuji, and Y. Iseri, Phys. Rev. C **67** (2003) 061001-R.
5. S. Ishikawa, M. Tanifuji, and Y. Iseri, Proc. of the Seventeenth International IUPAP Conference on Few-Body Problems in Physics. Durham, North Carolina, USA, June 5-10, 2003, edited by W. Glöckle, W. Tornow, (Elsevier, 2004) S61.
6. S. A. Coon, M. T. Peña, and D. O. Riska, Phys. Rev. C **52** (1995) 2925.
7. J. Adam, Jr., M. T. Peña, and A. Stadler, Phys. Rev. C **69** (2004) 034008.
8. C. M. Maekawa and M. R. Robilotta, Phys. Rev. C **57** (1998) 2839.
9. C. M. Maekawa, J. C. Pupin, and M. R. Robilotta, Phys. Rev. C **61** (2000) 064002.
10. R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51** (1995) 38.
11. H. T. Coelho, T. K. Das, and M. R. Robilotta, Phys. Rev. C **28** (1983) 1812.
12. M. R. Robilotta and H. T. Coelho, Nucl. Phys. **A460** (1986) 645.
13. A. Kievsky, Phys. Rev. C **60** (1999) 034001.
14. S. Ishikawa, in preparation.
15. R. D. Foster, *et al.*, Phys. Rev. C **73** (2006) 034002.
16. S. Ishikawa, M. Tanifuji, and Y. Iseri, Phys. Rev. C **64** (2003) 024001.